Research on Estimating the Variability of Purchased Highway Transportation Capacity

with Respect to Volume*

Michael D. Bradley
Department of Economics
George Washington University
Washington, D.C. 20052

* This research was supported by a grant from the United States Postal Service to the George Washington University. Professor Bradley thanks Daniel A. Broxterman, Karl Garbacik and Trenton Jin for excellent research assistance.

TABLE OF CONTENTS

A. Introduction 1

B. Investigating the Use of Operational Data to Estimate a Capacity-to-Volume Variability 4

C. Investigating the Commission's Docket No. N2010-1 Analysis 6

D. Refining and Extending the Commission's Approach to Estimating Capacity-to-Volume Variabilities 13

E. Eliminating TRACS Tests with Zero Volume 18

F. An Alternative Approach to Measuring Capacity 20

G. Testing and Correcting for Autocorrelation 23

H. Dropping FY2010 Data from the Analysis 29

I. Investigating an Alternative Time Aggregation 31

J. The Impact of the Capacity-to-Volume Variabilities on Attributable Costs 33

Mathematical Appendix 39

A. Introduction

The attributable costs for purchased highway transportation, like most cost components, are found though two steps, "attribution" and "distribution." Attribution is the measurement of the total volume variable (or attributable) costs in the cost component, and distribution is the assignment of those attributable costs to the individual products that cause them.

In many cost components, a single variability is used for finding the total attributable costs. That variability measures the rate at which the component cost responds to changes in the associated cost driver, and is multiplied by the component's accrued costs to find the volume variable, or attributable, costs. However, in the area of purchased highway transportation, the variability is more complex, and it has two components: 1) the variability of cost with respect to capacity and 2) the variability of capacity with respect to volume. Calculation of the overall purchased highway transportation variability thus depends upon a two-step process. That two-step process has been recognized and described by the Commission:[1]

> The first step focuses on the elasticity of the cubic-foot-miles (CFM) of capacity purchased relative to a change in the overall volume of mail using the transportation segment being analyzed. The second step focuses on the elasticity of the cost of purchased transportation relative to a change in the cubic-foot-miles of capacity purchased.

The two-step process can be represented graphically by showing the flow of causality from volume to cost. Changes in volume first lead to changes in capacity, and those capacity changes then lead to change in cost:

[1] See, Postal Rate Commission, Opinion and Recommended Decision, Docket No. R2000-1 (November 13, 2000) at 169.

$$\boxed{\text{Volume}} \longrightarrow \boxed{\text{Capacity}} \longrightarrow \boxed{\text{Cost}}$$

This two-step process means that the variability (or elasticity) of cost with respect to volume is expressed mathematically as the product of two elasticities, one for each step:[2]

$$\varepsilon_{Cost,Volume} = \varepsilon_{Cost,Capacity} * \varepsilon_{Capacity,Volume}$$

The variability of cost with respect to capacity for the various components of purchased highway transportation were recently updated and accepted by, the Commission.[3] In contrast, the variability of capacity with respect to volume has never been studied and has been set to one hundred percent, by assumption. In a previous docket, the Commission explained the basis for this assumption and highlighted the fact that the assumption is not the "final answer":[4]

> The elasticity of the first step has not received extensive empirical analysis and has thus far been taken to be 1.0. Some observers tend to view 1.0 as an upper limit for an elasticity of this kind, although it is not, since a 10 percent increase in volume could lead to a greater-than-10 percent increase in the CFM of capacity purchased. The use of 1.0 for this elasticity is based on descriptions of postal operations and contracting practices. These descriptions suggest that a CFM-of-capacity figure is developed as a requirement based on such things as length of contract, peak loads, volume fluctuations, and the nature of round trip contractor runs. The argument is that if there is a 10 percent

[2] In this circumstance, the terms "variability" and "elasticity" refer to the same mathematical formula and can be used interchangeably.
[3] See, Docket No. RM2014-6, Proposal Six, Order No. 2180, September 10, 2014.
[4] See, Postal Rate Commission, Opinion and Recommended Decision, Docket No. R2000-1 at 169.

> increase in overall postal volume, there will need to be an increase in the CFM-of-capacity purchased of about 10 percent. The Commission does not view this as a final answer; certain questions about it have been raised in this case.

The assumption of proportionality between volume and capacity implies that there are virtually no aspects of the Postal Services purchased highway transportation network that are determined by service requirements. In this view, when transported volume increases or decreases, the Postal Service maintains the same level of capacity utilization, and thus increases or decreases capacity in exact proportion to the volume change.[5] In other words, it is assumed that there is no fixity in the Postal Service purchased highway network, so increases in volume do not bring any opportunities to utilize capacity, and decreases in volume do not imply more challenges in using the required capacity.

The investigation of the validity of this assumption of proportionality between volume and capacity has taken on additional importance as the Postal Service has experienced ongoing volume declines. Casual empiricism suggests that capacity utilization may have fallen when volume was falling, raising the possibility that even if the proportionality assumption were correct in the past, it may no longer be so.

[5] For a mathematical demonstration of this condition, please see the Mathematical Appendix.

B. Investigating the Use of Operational Data to Estimate a Capacity-to-Volume Variability

We initially investigated estimating the capacity to volume variability using Postal Service operational data. The Postal Service records vehicle capacity and vehicle usage on a ”trip-leg” basis in its Transportation Information Management Evaluation System (TIMES) and its Surface Visibility (SV) data system. TIMES is a web-based data entry system that employs dock clerks to collect data about the volumes of mail arriving and leaving a mail processing facility on highway transportation. SV is an automated data collection system that makes use of bar codes and scanners to capture volume information at certain Postal Service facilities. At these locations, SV feeds TIMES.

To investigate the use of TIMES/SV data for estimating capacity-to-volume variabilities, we performed the following steps:

- We investigated the patterns and behavior of utilization in the purchased highway transportation network. This work involved construction of an analysis data set from the raw TIMES/SV data and matching it with routing information from National Air Surface System (NASS). That produced an analysis database that captured the regular transportation recorded in TIMES/SV that also was scheduled on the transportation frame. This set of regular transportation route-trip-legs was analyzed for patterns in utilization. This analysis illustrated the current usage of the network and supported subsequent analysis of the relationship between volume, utilization, and capacity.

- We examined the relationship between route-trip-legs and the listed facilities from which the route-trip-legs left and to which the route-trip-leg trips were destined. This work helped us focus on facility designation hygiene. It also generated an understanding of daily route schedules and provided insight into the use of frequency to provide moving capacity.

- We produced case studies of purchased highway transportation among a variety of facilities, to understand the relationship between volume, scheduled trips, frequency, and capacity. Each case study reviewed the associated route’s to and from facility pairs, listed trips, and combinations of route-trip-legs with its to/from pairs. Review of these case studies with Postal Service transportation experts provided the basis for subsequent analytical work that focused on variable creation and model specification.

- We worked, in consultation with Postal Service experts, to identify and construct an appropriate unit of observation for investigating the relationship between volume and capacity. This unit of observation must both be consistent with economic decision making by Postal Service transportation managers and be consistent with collected variables in existing Postal Service data systems.

- Most of the purchased highway transportation network is constructed using round trips from an origin facility to a destination facility with possible intermediate stops along the way. However, the TIMES/SV data is recorded at the route-trip-leg level with many local variations in the use of route, trip, and leg, assignments. This means that there is no straightforward way to construct a route-trip database from existing raw data. Consequently, we developed a matching logic to permit construction of unique route trips from the recorded TIMES/SV data. This was needed to support construction of the appropriate unit of observation for econometric analysis.

- We produced a sample analysis dataset. We used that data set to estimate some preliminary econometric regressions relating capacity to volume and to investigate the quality of the data.

The investigation of the TIMES/SV data produced substantial concerns with the quality of the TIMES/SV data for the purposes of measuring a capacity-to-volume variability. For example, the process of building the data set required a high amount of "data cleaning", which suggests a low level of reliability in the data. In addition, a number of key data quality issues arose, like difficulty in matching reported routings with building locations, concerns about the accuracy of the reported utilizations, missing observations for key variables like "leave date”, and some apparent irregularities in the operations of some routes. In sum, the data were not sufficiently reliable for supporting an econometric analysis.

C. Investigating the Commission's Docket No. N2010-1 Analysis

An important part of Docket No. N2010-1 was the estimation of potential cost savings accruing to the Postal Service resulting from the proposal to eliminate Saturday delivery. Some of those estimated cost savings were from reductions in the cost of purchased highway transportation.

In its primary analysis, the Commission accepted the Postal Service assertion that elimination of Saturday delivery would cause reductions in purchased highway transportation capacity needed on Saturday and Sunday. However, despite the existence of widespread empty space on the purchased highway transportation network on Monday through Friday, the Commission assumed that any saved capacity on Saturday and Sunday would be matched by additional capacity on the other days of the week. This approach implicitly assumed that there is exact proportionality between volume and capacity on a daily basis, so that shifting volume from one day to another did not result in savings in capacity.

The Commission also provided what it termed an "exploratory analysis," that attempted to estimate the relationship between volume and capacity, and thus produce elasticities of capacity with respect to volume.[6] This alternative approach to estimating purchased highway transportation cost savings then used these capacity-to-volume variabilities to calculate how much additional capacity would be needed on Monday through Friday, to absorb the transferred transportation volumes. The important point is that the Commission proposed a possible method of estimating the capacity-to-volume

[6] See, Postal Rate Commission, PRC-N2010-1-LR5, PRC Transportation Appendix, Docket No. N2010-1, (March 24, 2011).

variabilities needed for testing the assumption of proportionality built into the current method of calculating attributable purchased highway transportation costs.

The Commission's approach starts with an assumption that the number of truck trips is proportional to cubic foot-miles of capacity:[7]

> In general, changes in CF of transported mail can be expected to cause CFM [of capacity] to vary through changes in the number of truck trips. The latter can be expected to vary in the same direction as volume because of service-related concerns.

More specifically, the Commission's approach assumes the following relationship between cubic foot-miles and trips:

$$\frac{Trips}{CFM} = \rho\ .$$

The Commission's approach then specifies a "double log" model relating cubic foot-miles of capacity to volume:

$$\ln(CFM) = \alpha + \beta \ln(volume) + \varepsilon.$$

In this equation, β is the variability of capacity with respect to volume. The proportionality relationship, provided above, can be used to rewrite the log of cubic foot-miles:

$$\ln(CFM) = \ln(Trips) - \ln(\rho).$$

Substitution for ln(CFM) provides the following equation:

[7] Id., at 7.

$$\ln(Trips) = \ (\alpha + \ln(\rho)) \ + \ \beta \ln(volume) + \ \varepsilon.$$

Estimation of this equation provides an estimate of β, which is the elasticity of capacity with respect to volume.

To estimate this equation the Commission used TRACS data for 2005, 2008, 2009 and 2010.[8] It took the daily TRACS data and aggregated it by day-of-week, by quarter. This means that the data aggregation produced twenty-eight observations for the year, seven observations for each quarter. The Commission then estimated the above equation for four transportation types: Intra-SCF, Inter-SCF, Intra-BMC and Inter-BMC.[9]

The following table presents the estimated elasticities of capacity (trips) with respect to volume:

Table 1
PRC Estimated Trips Variability
Docket No.N2010-1

Account Type	**Estimated Trips Variability**
Intra-SCF	109.60%
Inter-SCF	66.79%
Intra-BMC	57.14%
Inter-BMC	68.56%

[8] The model estimated by the Commission also included dummy variables for the years 2008, 2009, and 2010. The estimated coefficients on these dummy variables were insignificant in most cases.

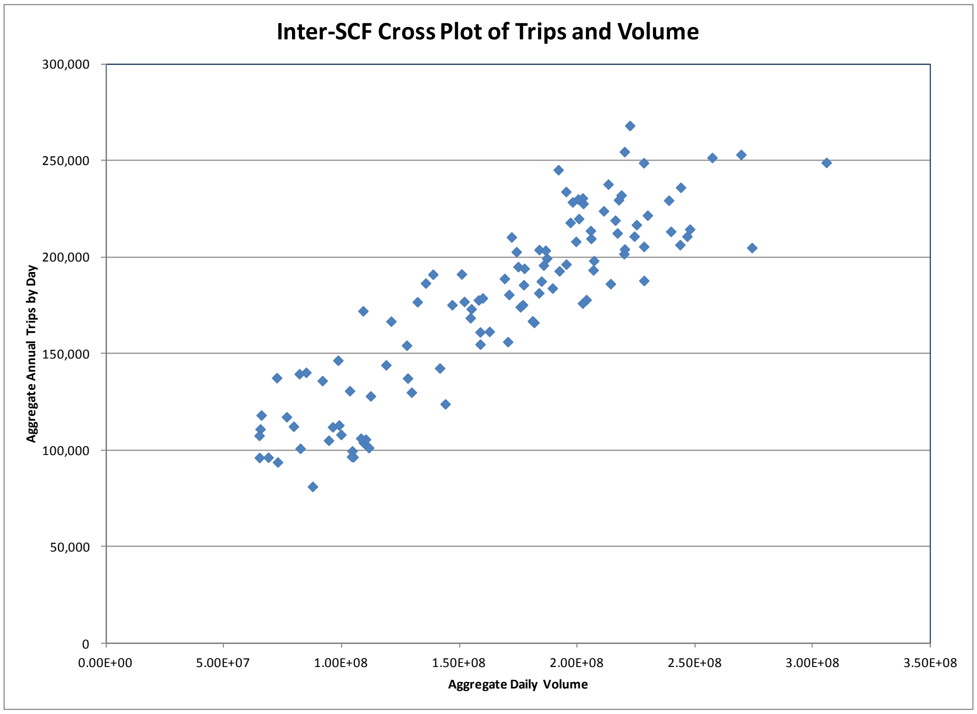
[9] The Postal Service has renamed BMCs as NDCs. Thus, it has renamed the latter two transportation account categories as Intra-NDC and Inter-NDC. For the balance of this report, we will use the more current names.

For the two NDC categories and the Inter-SCF category, the estimated elasticities are significantly less than one. These results contradict the assumption that capacity is proportional to volume. The Commission noted that the Intra-SCF variability was over 100 percent, but did not investigate whether this indicated an econometric problem. Instead, the Commission provided reasons why the unusual variability result could have been correct. It noted that the intra-SCF category had the lowest capacity utilization and suggested that the greater that 100 percent variability could come from "acute" service constraints:[10]

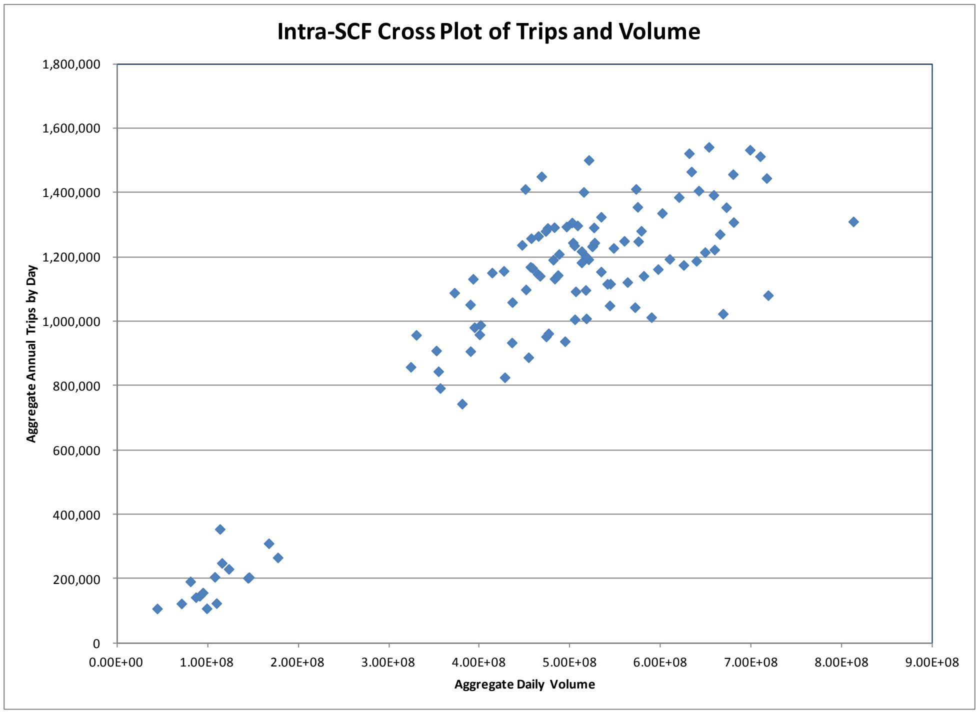
> When trip elasticity values are over 100 percent unused capacity increases as cubic feet increase because the number of trips increase disproportionately to volume. This type of sensitivity may be indicative of the acute service-related constraints that might be expected on intra-SCF transportation runs.

However, a careful look at the data reveals that the high variability for Intra-SCF transportation is actually a manifestation of the way TRACS data are collected. To see this, first consider a cross-plot of the trips and volumes for the Inter-SCF account category. Each dot in the following graph represents a day of week, postal quarter, fiscal year (e.g. Mondays, PQ1, FY 2011) observation on total TRACS trips and volumes.

[10] See, Postal Rate Commission, Advisory Opinion on Elimination of Saturday Delivery, Docket No. N2010-1 (March 24, 2011) at 99.



The plot shows the positive relationship between the two variables and shows that the relationship is consistent, as the data go from low-volume days to high-volume days. Now consider the same plot for the Intra-SCF account category, presented below. It presents a very different picture. The plot shows that there is a clear break in the data with a small subset of observations having very low volumes and numbers of trips compared to the other observations. Yet, within both the group of small observations and the group of larger observations, there appears to be the same positive relationship between trips and volume as exhibited by the Inter-SCF plot.



Investigation of the small trip and volume observations reveals that they occur on Sundays when there is much less Intra-SCF transportation. For example, testimony in Docket No. N2010-1 showed that Sunday transportation was a very small proportion of the total transportation for the Intra-P&DC and Intra-District accounts that make up the Intra-SCF grouping. In FY2009 (included in that dataset), Sunday accounted for just 4.2 percent of intra-P&DC transportation and just 0.6 percent of Intra-District transportation.[11]

Because there are fewer trips on Sunday, there are fewer TRACS tests taken on Sundays than on the other days of the week for the Intra-SCF account grouping. The following table presents the quarterly average number of TRACS tests by day of week in 2010 for the Intra-SCF account group.

[11] See, Direct Testimony of Michael D. Bradley on Behalf of the United States Postal Service, Docket No. N2010-1 (March 30, 2010) at 37.

Table 2
Quarterly Average Number of TRACS
Tests For Intra-SCF by DOW in FY 2010

Day of Week	Average
Sunday	13.0
Monday	103.8
Tuesday	108.3
Wednesday	89.0
Thursday	103.3
Friday	106.0
Saturday	94.5

When there are fewer tests taken, summing the number of trips and volume will lead to smaller magnitudes, even if the relationship between the number of trips and volume is the same as on days with more tests taken. Not accounting for this artificial difference in the size of the observations will lead to spurious regression results and is the cause of the estimated variability of more than 100 percent.

We can control for this data construction issue by including a dummy variable for the Sunday observations, and then estimating the equation for the Intra-SCF account grouping:

$$\ln(Trips) = \ (\alpha + \ln(\rho)) \ + \ \beta \ln(volume) + \delta \, D_S + \ \varepsilon,$$

where D_S = 1 for the small volume and trip observations that occurred on Sundays, and D_s = 0 for all other observations. When this variable is included in the equation, the results are quite a bit different. As expected, the estimated coefficient on the dummy variable is negative and significantly different from zero and, once the equation is adjusted for the different number of tests on Sunday, the variability falls to 59.8 percent.

This is in the same neighborhood as the other estimated variabilities and supports the conclusion that the variability of capacity with respect to volume is well less than 100 percent.

Table 3
Results for Intra-SCF Equation
Including a Size Dummy

Variable	Coefficient	t-statistic
Ds	-0.907	-7.960
B	0.598	8.927

D. Refining and Extending the Commission's Approach to Estimating Capacity-to-Volume Variabilities

The Commission's variability analysis provided substantial evidence that the elasticity of capacity with respect to volume is materially less than one hundred percent. However, further analysis is warranted to investigate the robustness of these results and to update them using more recent data. Updating the data helps ensure the estimated variabilities reflect the current purchased highway transportation structure.

To re-estimate and update the PRC equation, we obtained TRACS data for fiscal years 2010 through 2015. This dataset includes 56,369 tests covering the four account categories. We collected information on the cubic capacity, number of trips, and cubic volume of mail for each TRACS test. A typical observation from the dataset is presented in Table 4.

Note that trips, cubic capacity, and mail volume are all calculated at an annual frequency. This example test covered a route that runs three days a week except holidays yielding 154.759 trips per year. If one divides the annual cubic capacity

(410,110) by the number of trips, one gets the cubic capacity per trip of 2,650 cubic feet, the capacity of a 40-foot trailer.

Table 4
Example of a TRACS Data Set Observation

Variable	**Value**
Test ID	10119IA
Contract Type	INTER-NDC
Fiscal Year	2010
Postal Quarter	1
Day of Week	1
Annual Cubic Capacity	410,110
Annual Mail Volume Cube	246,066
Utilization	60%
Annual Trips	154.759

Table 5 provides the annual averages for each of the four account categories for each of the six years in the database. As expected, the Intra-SCF category has many more trips and much smaller capacities then the other three account categories. This is consistent with the nature of the local transportation and the need to make service. At the other end of the spectrum, the Inter-NDC category has the largest vehicles and fewest trips. This is consistent with long-haul transportation made up solely of tractor-trailers that often have a more relaxed dispatch window.

Table 5
Average Values, By Contract Type and Year, from TRACS Data

Contract Type	Fiscal Year	Number of Tests	Average Annual Number of Trips per Contract	Average Cubic Capacity Per Trip
INTRA-SCF	2010	2,471	11,875.1	1,338.1
INTRA-SCF	2011	2,835	9,911.4	1,325.4
INTRA-SCF	2012	2,804	9,759.1	1,345.3
INTRA-SCF	2013	2,697	9,826.7	1,374.9
INTRA-SCF	2014	2,716	10,309.8	1,433.6
INTRA-SCF	2015	2,669	10,269.8	1,382.7
INTER-SCF	2010	2,397	2,097.5	2,264.3
INTER-SCF	2011	2,706	1,635.7	2,222.9
INTER-SCF	2012	2,727	1,572.4	2,279.2
INTER-SCF	2013	2,570	1,665.5	2,314.9
INTER-SCF	2014	2,566	1,822.7	2,276.7
INTER-SCF	2015	2,535	2,032.0	2,264.7
INTRA-NDC	2010	2,097	686.9	2,906.5
INTRA-NDC	2011	2,096	560.6	2,900.8
INTRA-NDC	2012	2,086	622.6	2,970.2
INTRA-NDC	2013	1,975	634.8	2,921.3
INTRA-NDC	2014	2,017	637.6	2,930.4
INTRA-NDC	2015	1,962	645.1	2,938.6
INTER-NDC	2010	2,071	223.3	3,059.5
INTER-NDC	2011	2,135	179.4	3,104.8
INTER-NDC	2012	2,113	167.2	3,117.1
INTER-NDC	2013	2,055	190.6	3,142.3
INTER-NDC	2014	2,080	196.5	3,127.1
INTER-NDC	2015	1,989	218.6	3,146.4

Finally, note the numbers of trips were all much larger in 2010 than in the subsequent years. There was about a 20 percent decline from 2010 to 2011, with recovery in 2014 and 2015. The disparity between the number of trips in 2010 and the subsequent years suggests that one should allow for possible shift in size of the dependent variable in the estimated econometric equations.

The 56,369 TRACS observations were summed by fiscal year, postal quarter and day of the week to produce 168 observations on capacity, volume, and trips for each of the four account categories.[12] These more recent data were then used to estimate the equation measuring the relationship between the number of trips and volume.

In addition, three refinements were made to the model. First, a dummy variable for FY 2010 was included to control for the size shift between FY 2010 and subsequent years. Not controlling for this change could lead to a misestimated variability. Next, a second order term in the log of volume was included to allow for non-linearity in the relationship between trips and volume. This converts the model into a simple trans-log specification. Finally, a day of week variable was included to check whether the weekly volume pattern has an influence on the volume-to-capacity relationship.

With these additions, the equation estimated is given by:[13]

$$\ln(Trips) = \beta_0 + \beta_1 \ln(Volume) + \beta_2 \ln(Volume)^2$$
$$+\delta_1 D_{2010} + \delta_2 D_S + \delta_3 DOW + \varepsilon$$

The full results of estimating this equation for each of the four account categories are provided in USPS-RM2016-12/1, but the resulting variabilities are presented below.

[12] Recall that the data are organized by the day of the week. Consequently, there are seven observations per quarter and 28 observations per year. Six years of data thus provide 168 observations.
[13] The size dummy variable (D_S) had a non-zero coefficient only for the Intra-SCF account category.

The dummy variable for 2010 was significant in all specifications, indicating there was an important level shift between 2010 and the subsequent years. In contrast, the day of week variable was significant only for the Inter-NDC account type. Finally, the second order term in the log of volume was significant only for the Inter-SCF account type.

Table 6
Estimated Trips Variabilities Using FY2010 through FY2015 Data

Account Type	Double Log Model	Translog Model
Intra-SCF	38.75%	46.85%
Inter-SCF	74.11%	69.55%
Intra-NDC	57.77%	61.13%
Inter-NDC	75.32%	72.97%

The estimated variabilities are well below one, as they were in the Commission's original analysis. The results thus strongly indicate that the variability of capacity with respect to volume is well below one hundred percent. In fact, one can test the proportionality hypothesis by testing whether or not the estimated elasticities are significantly different from one. Such an exercise requires testing the following null hypothesis:

$$H_o\colon \beta_1 = 1; \quad H_1\colon \beta_1 \neq 1.$$

The hypothesis is tested with an F-test of the restriction that $\beta_1 = 1$.

The next table presents the calculated test statistics for testing this null hypothesis. In all instances, the hypothesis is strongly rejected.

Table 7
F-tests of the Null Hypothesis $\beta_1 = 1$

Account Type	Calculated Test Statistic	Probability Value
Intra-SCF	243.97	<.0001
Inter-SCF	43.29	<.0001
Intra-NDC	257.26	<.0001
Inter-NDC	76.28	<.0001

The updated results also reflect the pattern of results found in the earlier data. Both "intra" elasticities are less than there corresponding "inter" elasticities, perhaps reflecting their low utilization rates. In addition, the SCF elasticities are lower than the NDC elasticities for the same type of transportation, intra vs. inter. This pattern may reflect that service requirements impose more network fixity on the SCF portion of the network than on the NDC portion of the network.

E. Eliminating TRACS Tests with Zero Volume

Another possible refinement of the Commission's model merits consideration. A material proportion of TRACS tests identify trucks that contain no volume at the time of the test. In these cases, the data set includes the cubic capacity and number of trips for the route on which the test was taken, but no volume. When the individual tests are aggregated to the daily totals, the trips from these tests are included without any accompanying volume. This potential mismatch could cause the data to understate the true relationship between the number of trips and volume and thus cause the estimated equations to understate the variabilities.

To investigate this issue we recalculated the daily totals after excluding the tests with zero volume. Note the number observations used to estimate the equations remains the same, 168. The difference is that the numbers of trips in those observations will be reduced as those with zero volume trucks are eliminated. The full results of this estimation exercise are presented in USPS-RM2016-12/1, but the variabilities for the four different types of contracts are presented in Table 8.

Table 8
Estimated Trips Variabilities Using FY2010 through FY2015
Data and Dropping Zero Volume Tests

Account Type	Double Log Model	Translog Model
Intra-SCF	60.42%	57.08%
Inter-SCF	83.92%	76.97%
Intra-NDC	75.35%	77.83%
Inter-NDC	85.06%	81.18%

Increases From Corresponding Variabilities Including Zero
Volume Tests

Account Type	Double Log Model	Translog Model
Intra-SCF	21.67%	10.23%
Inter-SCF	9.81%	7.42%
Intra-NDC	17.58%	16.70%
Inter-NDC	9.74%	8.21%

In all cases, there are substantial increases in the estimated variabilities, as expected. Nevertheless, none of the estimated variabilities approaches 100 percent, so these results also reject the assumption of proportionality.

F. An Alternative Approach to Measuring Capacity

The Commission's original approach to estimating a variability of capacity with respect to volume uses trips as its measure of capacity. To do so, the Commission is assuming proportionality between trips and cubic foot-miles. To gain a bit more understanding of this assumption, recall that cubic toot-miles has three components, the cubic capacity of the vehicle (Cube), the frequency at which the vehicle runs (Trips) and the route miles that the vehicle traverses (Miles):

$$CFM = (Cube * Trips * Miles)$$

Because the Commission's approach is assuming proportionality between trips and cubic foot-miles, at least in response to volume changes, it is necessarily assuming that neither cubic capacity nor route miles change when volume changes.[14] In terms of route miles, this seems like a reasonable assumption. Route miles are primarily determined by facility location and play little or no role in the determination of the elasticity of capacity with respect to volume. The capacity created by a 3800 cube trailer making two 50-mile trips is identical to the capacity created by a 3800 cube trailer

[14] As explained above, the Commission's original approach assumes that trips are a constant proportion (ρ) of cubic foot-miles or, $(Trips/CFM) = \rho$. Substituting the definition of cubic foot-miles and cancelling terms yields: $(1/(Cube * Miles)) = \rho$. This formulation shows that the Commission's original approach is holding cube constant in addition to miles.

making one 100-mile trip.[15] On the other hand, the assumption of proportionality is more difficult to accept in terms of cubic capacity. As volume rises or falls, it is possible for the Postal Service to adjust the sizes of the trucks that it runs, particularly for local transportation, in which there are many different sized trucks in use. If the Postal Service does adjust truck size, then the use of the number of trips as a measure of capacity may lead to an understated variability of capacity with respect to volume. In essence, the assumption is precluding a response in a possibly important aspect of capacity.

To investigate this possibility, one could use a broader measure of capacity in the variability equation for capacity with respect to volume. The dependent variable in that equation could be "moving capacity" which is cubic capacity multiplied by trips. This approach relaxes the assumption that cubic foot-miles are proportional to the number of trips and allows moving capacity to respond to volume changes in two ways, changes in trips and changes in cubic capacity. The resulting econometric equation is:

$$\begin{aligned}\ln(Capacity) \quad &= \lambda_0 + \lambda_1 \ln(Volume) + \lambda_2 \ln(Volume)^2 \\ &\quad + \delta_1 D_{2010} + \delta_2 D_S + \delta_3 DOW + \varepsilon\end{aligned}$$

The complete set of regression results are presented in USPS-RM2016-12/1, but Table 9 provides the estimated elasticities.

[15] The cost of these two routes is not necessarily the same, however. When measuring the elasticity of cost with respect to capacity, route miles played an important role. This is because of the characteristic of unit highway transportation costs to decline with miles travelled (the "distance taper").

Table 9
Estimated Moving Capacity Variabilities Using FY2010 through FY2015
Data and Dropping Zero Volume Tests

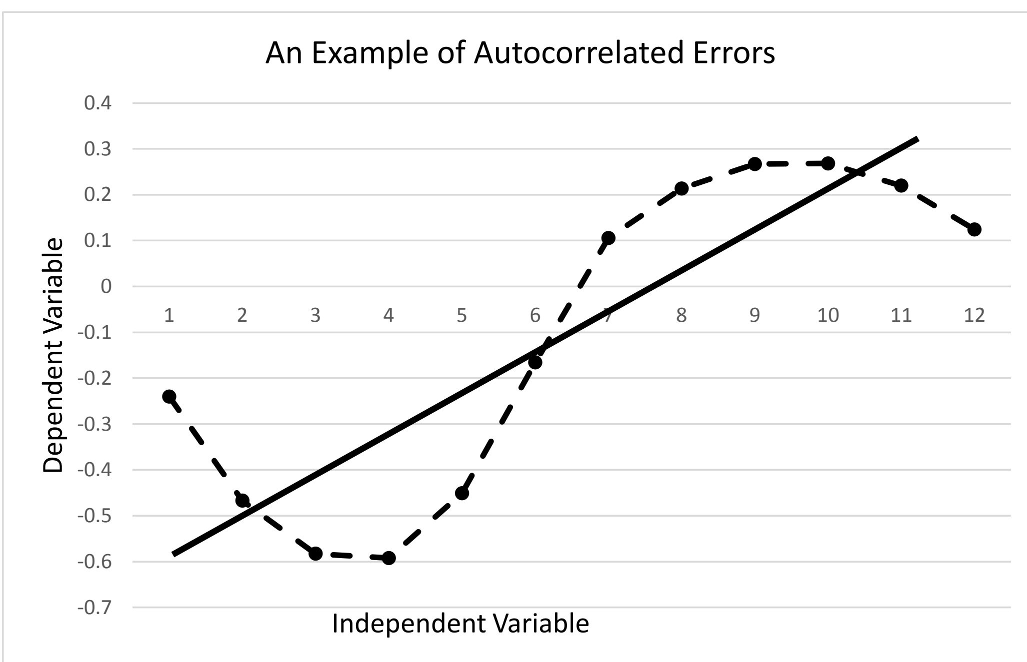
Account Type	Double Log Model	Translog Model
Intra-SCF	70.26%	75.12%
Inter-SCF	79.29%	77.94%
Intra-NDC	74.92%	77.97%
Inter-NDC	85.87%	82.44%

The estimated capacity variabilities for the Inter-SCF, Intra-NDC, and Inter-NDC are similar to the respective trips variabilities, with some a bit higher and some a bit lower. The exception is for the Intra-SCF capacity variabilities that are quite a bit above the trips variabilities. The variability from the double log model is 9.84 percentage points higher when the zero volume tests are dropped and the translog variability increases by 18.04 percentage points after dropping the zero volume tests (refer to Table 8). Such an outcome make sense, because Intra-SCF transportation has the lowest percentage of tractor trailer routes and the highest possibilities for changing truck size, up or down, in response to volume changes. Thus, the Intra-SCF category is most likely to be affected by a change in the definition in capacity that allows for variations in truck size. In sum, the result with the alternative approach to measuring capacity indicate that it is preferred to use the broader definition of capacity when estimating volume to capacity variabilities.

G. Testing and Correcting for Autocorrelation

A basic assumption of the standard regression model is that the error terms are not correlated with one another. However, in time series data, it can often be the case that the error terms are related to each other through time. For example, agricultural data may have correlated error terms if the effect of a drought is felt in more than one month. Suppose that an unexpected drought causes output to be below what would be normally expected for a month. Then, there would be a negative error term for that month. If the effects of the drought linger, then error terms for subsequent months could also be negative, pattern that produce a correlation among the error terms. A similar pattern could occur for positive shocks, when unexpectedly good weather occurs and output rises above its expected level.

A key point is that error terms will be persistently away from the regression line, in an identifiable pattern. In this instance, the error terms are said to be autocorrelated or serially correlated. For example, the next graph presents a picture of positively autocorrelated errors.



Autocorrelation does not affect the parameter estimates of the regression model, which remain unbiased and consistent. Thus, autocorrelation does not affect the estimated coefficients that measure the capacity-to-volume variabilities. It does, however, affect their variances, and the inferences that are based upon those variances. If t-tests and F-tests are used for model specification, autocorrelation can render those tests inaccurate. In addition, autocorrelation can provide a warning that an important variable has been omitted from the regression equation. Most economic time series data are correlated through time. If an important variable is omitted from the equation, then its effect will show up in the regression's residuals. Because that omitted variable is likely correlated with itself through time, the residual from an equation with an important omitted variable will also be correlated through time and this will show up as autocorrelation. Because true autocorrelation does not affect the parameter estimates, correction for autocorrelation should not have a large impact on those estimates. If it does, then it is possible that the observed autocorrelation is actually reflecting an important omitted variable.

We employ two tests for autocorrelation. The first is the Durbin-Watson statistic. The statistic has the following formula:

$$d = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2}.$$

The Durbin-Watson d statistic is approximately equal to 2(1-r), where r is the autocorrelation coefficient of the residuals. If there is no autocorrelation, then r = 0 and d = 2. If there is significant positive autocorrelation, then r > 0 and the calculated d statistic is well less than 2 with a lower bound of 0. In contrast, if there is significant negative autocorrelation, then r < 0 and the calculated d statistic is well larger than 2, with an upper bound of 4. To test for autocorrelation one compares the calculated d statistic with both upper and lower critical values. Table 10 presents the Durbin-Watson d statistics for each of the equations along with the probability values for positive and negative autocorrelation. The calculated test statistics indicate that autocorrelation is a potential problem for Inter-NDC equations, is not likely to be a problem for Inter-SCF and Intra-NDC equations, and is not a problem for Intra-SCF equations.

Table 10
Testing for Autocorrelation Using the Durbin Watson Statistic

Transportation Type	Model	DW Statistic	Probability Value Testing for Positive Autocorrelation	Probability Value Testing for Negative Autocorrelation
Intra-SCF	Double Log	2.013	0.4920	0.5080
Intra-SCF	Translog	2.025	0.5211	0.4789
Inter-SCF	Double Log	1.800	0.0778	0.9222
Inter-SCF	Translog	1.786	0.0643	0.9357
Intra-NDC	Double Log	1.792	0.0693	0.9307
Intra-NDC	Translog	1.802	0.0791	0.9209
Inter-NDC	Double Log	1.283	0.0001	1.0000
Inter-NDC	Translog	1.303	0.0001	1.0000

While the Durbin Watson statistic can indicate the presence of autocorrelation, it does not provide any guidance about the order of that autocorrelation. That is, it does not help determine how many lags the correction for autocorrelation should include. To investigate the order of the autocorrelation correction, we calculate the Godfrey Lagrange Multiplier (LM) Tests. This is a Chi-Squared test for the presence of significant autocorrelation terms and can be performed for any number of lags. Moreover, each of the individual test statistics has power against different alternative hypotheses.

If the residuals have first order autocorrelation, then the one-lag test is most likely to reject the null of no autocorrelation. If the residuals have second order autocorrelation, then the two-lag test is most likely to reject the null of no autocorrelation. Investigation of the series of LM tests indicates the appropriate order to include in the autocorrelation correction. Table 11 shows that the Godfrey test indicates that autocorrelation is clearly an issue for the Inter-NDC equation, as that type of transportation has a set of test statistics that indicate rejection of the null hypothesis of no autocorrelation. In contrast, the Durbin-Watson statistics indicate that the Intra-SCF equation does not suffer from autocorrelation and that result is confirmed by the Godfrey test.

The Durbin-Watson statistics also indicate rejection of autocorrelation for the Inter-SCF equation, although the values are relatively close to the threshold. But the Inter-SCF Godfrey statistics are far from the critical values needed to reject the null hypothesis of no autocorrelation. This combination suggests that autocorrelation is not likely present. To check this inference, the Inter-SCF was estimated with an autocorrelation correction, but the estimated autocorrelation parameter is not statistically significant. From this combination of results, we can conclude that autocorrelation is not present in the Inter-SCF equation.

Table 11

Testing for Autocorrelation Using Godfrey's Lagrange Multiplier Statistic

Intra-SCF

Double Log			Translog		
Alternative	LM Statisitc	Probability Value	Alternative	LM Statisitc	Probability Value
AR(1)	0.016	0.9002	AR(1)	0.039	0.8445
AR(2)	0.021	0.9897	AR(2)	0.068	0.9665
AR(3)	0.084	0.9937	AR(3)	0.133	0.9877
AR(4)	0.117	0.9984	AR(4)	0.145	0.9975
AR(5)	0.419	0.9948	AR(5)	0.560	0.9898
AR(6)	0.420	0.9987	AR(6)	0.632	0.9958

Inter-SCF

Double Log			Translog		
Alternative	LM Statisitc	Probability Value	Alternative	LM Statisitc	Probability Value
AR(1)	1.661	0.1975	AR(1)	1.964	0.1611
AR(2)	1.989	0.3700	AR(2)	2.344	0.3097
AR(3)	1.990	0.5744	AR(3)	2.351	0.5029
AR(4)	2.286	0.6834	AR(4)	2.712	0.6071
AR(5)	4.768	0.4449	AR(5)	5.484	0.3597
AR(6)	5.232	0.5144	AR(6)	5.903	0.4342

Intra-NDC

Double Log			Translog		
Alternative	LM Statisitc	Probability Value	Alternative	LM Statisitc	Probability Value
AR(1)	1.674	0.1957	AR(1)	1.482	0.2234
AR(2)	1.800	0.4066	AR(2)	1.545	0.4618
AR(3)	7.924	0.0476	AR(3)	7.780	0.0508
AR(4)	8.456	0.0762	AR(4)	8.470	0.0758
AR(5)	9.353	0.0958	AR(5)	9.116	0.1045
AR(6)	9.625	0.1414	AR(6)	9.288	0.1580

Inter-NDC

Double Log			Translog		
Alternative	LM Statisitc	Probability Value	Alternative	LM Statisitc	Probability Value
AR(1)	21.517	0.0001	AR(1)	20.406	0.0001
AR(2)	30.884	0.0001	AR(2)	29.399	0.0001
AR(3)	38.193	0.0001	AR(3)	36.698	0.0001
AR(4)	41.998	0.0001	AR(4)	40.109	0.0001
AR(5)	42.775	0.0001	AR(5)	40.674	0.0001
AR(6)	43.187	0.0001	AR(6)	41.352	0.0001

A different pattern holds for the Intra-NDC equations. Although the Durbin-Watson statistics indicate a rejection of the presence of autocorrelation, they are on the borderline. However, the Godfrey statistic is close to statistical significance at the fourth lag and is significant at lag three. As are result, like the Inter-SCF equation, the Intra-NDC model was estimated with an autocorrelation correction included. That effort produced statistically significant autocorrelation coefficients, so we will apply an autocorrelation correction to that transportation category. Investigation of the estimated autocorrelation coefficients showed that third order was the highest order with a statistically significant coefficient, so a third-order correction was applied.

Table 12 presents the variability coefficients and their associated standard errors, both before and after correcting for autocorrelation.[16] There is very little difference between the corrected and uncorrected coefficients and standard errors, signifying that neither autocorrelation nor omitted variables are a material problem for these estimations.

[16] Recall that in both the double log and translog models, the variability is measured by the coefficient on volume.

Table 12
Impact of Autocorrelation Correction on Estimated Variabilities and Standard Errors

Transportation Type	Model	Pre Correction Parameter Estimate	Pre Correction Standard Error	Post Correction Parameter Estimate	Post Correction Standard Error
Intra-NDC	Double Log	0.7492	0.0247	0.7638	0.0234
Intra-NDC	Translog	0.7797	0.0304	0.7910	0.0285
Inter-NDC	Double Log	0.8587	0.0264	0.8456	0.0216
Inter-NDC	Translog	0.8244	0.0345	0.8274	0.0295

H. Dropping FY2010 Data from the Analysis

As discussed above, review of the raw data indicated that FY2010 experienced a different level of purchased highway transportation than did subsequent years. The number of trips fell by double-digit percentages after 2010. To account for this shift, the estimated models include a level-shift dummy for 2010 and in most instances, the estimated coefficient on the dummy was statistically significant. This confirms that, for purposes of estimating a capacity-to-volume variability, FY2010 was different.

Because we have multiple years of data with which to estimate the variability, it is possible to estimate the various models using data just from the FY2011 through FY2015. This approach eliminates any possibility that the estimated variabilities could be affected by data from a transportation network that no longer exists.

The equations for all four categories were re-estimated using the FY2011 through FY2015 data. Despite the reduction in the number of observations (to 140), the models fit the data well and provided sensible results. Only the Inter-NDC equations were subject to autocorrelation, and the correction was applied.[17] Table 13 provides the variabilities estimated on the more recent data, along with the corresponding variabilities, previously estimated, that used all of the data. The table reveals that dropping the FY2010 data caused an increase in the estimated variabilities for the translog model, except for the Intra-NDC category, which was virtually the same, and little change in the double log model.

Table 13
Estimated 2011-2015 Capacity Variabilities, Dropping Zero Volume Tests and Correcting Inter-NDC for Autocorrelation

Double Log Model

Transportation Type	FY2010-FY2015[1]	FY2011-FY2015	Change
Intra-SCF	0.7026	0.7076	0.0050
Inter-SCF	0.7929	0.8009	0.0080
Intra-NDC	0.7638	0.7531	-0.0107
Inter-NDC	0.8456	0.8584	0.0128

Translog Model

Transportation Type	FY2010-FY2015[1]	FY2011-FY2015	Change
Intra-SCF	0.7512	0.7727	0.0215
Inter-SCF	0.7794	0.8212	0.0418
Intra-NDC	0.7910	0.7877	-0.0033
Inter-NDC	0.8274	0.8482	0.0208

[1]The FY2010-FY2015 Intra-NDC Variabilities are corrected for autocorrelation and are taken from the Post Correction column of Table 12.

[17] After dropping the FY2010 data, both the Durbin-Watson and Godfrey tests indicated that there was no autocorrelation for the Intra-SCF, Inter-SCF and Intra-NDC equations. Full results are presented in USPS-RM2016-12/1.

I. Investigating an Alternative Time Aggregation

The empirical approach suggested by the Commission aggregates, by day of week, data from all of the TRACS tests for postal quarter. In other words, the observation for Monday in FY 2011, quarter 2, includes all the TRACS tests done on Mondays in that quarter. This is done for each of the four transportation types.

This approach has the strength of including many individual TRACS tests in each observation, and it averages 83.9 TRACS tests (68.7 if the zero volume observations are dropped) for each weekday observation. However, it has the weakness of producing just 7 observations per quarter or just 28 observations per year. Moreover, the constructed time series is not a natural series in the sense of being regular observations at a known frequency like week, month, or quarter.

To check the robustness of this particular time aggregation, we pursued an alternative, more standard approach. In this alternative approach, the TRACS tests were cumulated by week. A week was chosen for time aggregation because it still provides a substantial number of TRACS tests per observation (averaging 44.5 for all observations and 36.4 with zero volume observations dropped), but increases the number of observations per year to 52. This increases the number of observations available to estimate the variability equation to 312 from 168.

To provide comparable results with the previous method of time aggregation, we estimated a set of models with moving capacity as the dependent variable, using data from FY2010 through FY2015, dropping zero volume tests and, where appropriate, correcting for autocorrelation. One slight difference in specification was required for estimating the models on weekly data. With the reduction of the number of TRACS tests per observation, the possibility of observations with a small number of tests (less than 20) increases. A review of the individual observations revealed that there are now a few observations for Inter-SCF, Intra-NDC, Inter-NDC categories, as well as the Intra-SCF category that have a small number of tests. Consequently, the dummy variable that has been used for the Intra-SCF model is now included in the models for the other three categories. Table 14 presents the variabilities estimated on the weekly data. Note that the second order volume term was significant only for the Inter-SCF category and the autocorrelation correction was required only for the Inter-NDC equation.

Table 14
Estimated Capacity Variabilities Using Weekly Data From FY2010 through FY2015 and Dropping Zero Volume Tests

Account Type	Double Log Model	Translog Model
Intra-SCF	72.92%	71.25%
Inter-SCF	73.90%	74.95%
Intra-NDC	75.32%	72.63%
Inter-NDC	83.20%	81.60%

The variabilities estimated on the weekly data are quite similar to those estimated on the day-of-week data (Table 9). Of the eight estimated variabilities only the Intra-SCF double log model has a higher value when weekly data were used. The weekly trans-log model for Intra-SCF produces a variability about 4 percentage points lower than the variability produced by the day-of-week model. For the other three accounts, the weekly variabilities were all lower, averaging a decline of about 3 percentage points.

The main point to be drawn by this exercise is that all of the estimated variabilities are well less than one hundred percent and are close in value to those estimated on the day of week data. This demonstrates that the day of week variabilities are robust and that the resulting variabilities are not an artifact of the method of constructing the data.

J. The Impact of the Capacity-to-Volume Variabilities on Attributable Costs

As explained in the introduction, the variability of purchased highway costs has two parts, the variability of cost with respect to capacity, and the variability of capacity with respect to volume. Because the latter variability has been assumed to be one hundred percent, the overall variability of purchased highway transportation (by transportation type) has historically been just the variability of cost with respect to capacity. We now can insert the estimated capacity-to-volume variabilities into the formula, so the overall variability, for each transportation time, is the product of the two individual variabilities.

The capacity-to-volume variabilities that will be applied come from Table 13. They are from the translog model using day-of-week data from FY2011 through FY2015, corrected for autocorrelation. Although the second order terms are not significant for three of the estimated equations, they appear to be helping to more accurately estimate the first-order term, as in all cases, the translog model provides a variability that is higher than the double log model, often by several percentage points. In addition, using the versions of the models that exclude the FY2010 data provides variabilities that more closely mirror the current purchased highway transportation network.

Table 15 presents the current cost-to-capacity variabilities along with their associated capacity-to-volume variabilities. Note that the two categories that make up Intra-SCF transportation, (Intra-P&DC and Intra-District) and the three categories that make up Inter-SCF transportation (Inter-P&DC, Inter-Cluster, and Inter-Area) all have the same capacity-to-volume variability.

Table 15
Calculating the Overall Purchased Highway Transportation Variabilities

Transportation Category	Cost to Capacity Variability	Capacity to Volume Variability	New Overall Variability
INTRA P&DC	75.7%	77.3%	58.5%
INTRA DISTRICT	38.0%	77.3%	29.4%
INTER-SCF	89.1%	82.1%	73.2%
INTER P&DC	85.0%	82.1%	69.8%
INTER CLUSTER	89.1%	82.1%	73.2%
INTER AREA	89.9%	82.1%	73.8%
INTRA-NDC	94.9%	78.8%	74.7%
INTER-NDC	94.7%	84.8%	80.3%

Because of the multiplicative relationship between the two individual variabilities, and because both sets of variabilities are less than one, the product of the two variabilities will necessarily be below the previous overall variability. This means that application of the new capacity-to-volume variabilities will have the effect of reducing attributable purchased highway transportation costs. There is some variation in the estimated capacity-to-volume variabilities, with the Intra-SCF variability being the lowest and the Inter-NDC variability being the highest. This variation means the impact on individual product attributable transportation costs will depend upon the mix of purchased highway transportation cost used by the product. A product that predominantly uses Intra-SCF transportation will experience a larger proportional reduction in attributable transportation cost than a product that primarily uses Inter-NDC transportation.

The impacts of the estimated capacity-to-volume variabilities on attributable transportation costs, by market dominant product, are presented in Table 16. The analogous impacts for competitive products are presented in USPS-RM2016-12/NP1. Note that these costs include all purchased transportation costs, not just purchased highway costs. Thus, a product that also materially uses air transportation will see a smaller percentage decline in costs, because air transportation costs are not affected. That explains why the percentage reduction in First-Class attributable transportation costs is smaller than the percentage reduction in Standard Mail attributable transportation costs.

Table 16

Impact of Capacity to Volume Variabilities on FY 2015 Transportation Costs ($000s)

	New Total Transportation Cost	Old Total Transportation Cost	Absolute Change	Percentage Change
FIRST-CLASS				
SINGLE-PIECE LETTERS	$245,391	$287,434	-$42,043	-14.6%
SINGLE-PIECE CARDS	$3,626	$4,351	-$725	-16.7%
PRESORT LETTERS	$442,532	$481,192	-$38,660	-8.0%
PRESORT CARDS	$13,637	$15,163	-$1,526	-10.1%
SINGLE PIECE FLATS	$145,407	$165,495	-$20,087	-12.1%
PRESORT FLATS	$56,736	$62,634	-$5,898	-9.4%
PARCELS	$50,626	$59,307	-$8,681	-14.6%
TOTAL FIRST-CLASS	**$957,956**	**$1,075,576**	**-$117,620**	**-10.9%**
STANDARD MAIL				
HIGH DENSITY AND SATURATION LETTERS	$4,961	$6,160	-$1,199	-19.5%
HD& SATURATION FLATS & PARCELS	$17,798	$22,248	-$4,450	-20.0%
CARRIER ROUTE	$42,609	$52,410	-$9,801	-18.7%
LETTERS	$161,345	$193,039	-$31,694	-16.4%
FLATS	$148,809	$179,127	-$30,318	-16.9%
PARCELS	$7,084	$8,544	-$1,461	-17.1%
EVERY DOOR DIRECT MAIL - RETAIL	$0	$0	$0	0.0%
TOTAL STANDARD MAIL	**$382,606**	**$461,529**	**-$78,923**	**-17.1%**
PERIODICALS				
IN COUNTY	$98	$123	-$26	-20.8%
OUTSIDE COUNTY	$188,782	$226,850	-$38,069	-16.8%
TOTAL PERIODICALS	**$188,880**	**$226,974**	**-$38,094**	**-16.8%**
PACKAGE SERVICES				
ALASKA BYPASS	$15,860	$19,147	-$3,288	-17.2%
BOUND PRINTED MATTER FLATS	$21,054	$25,539	-$4,485	-17.6%
BOUND PRINTED MATTER PARCELS	$20,101	$24,503	-$4,401	-18.0%
MEDIA AND LIBRARY MAIL	$85,807	$102,420	-$16,612	-16.2%
TOTAL PACKAGE SERVICES	**$142,823**	**$171,609**	**-$28,786**	**-16.8%**
FREE MAIL - BLIND, HANDICAPPED, AND SERVICEMEN	**$5,173**	**$5,934**	**-$761**	**-12.8%**
TOTAL DOMESTIC MARKET DOMINANT	**$1,704,064**	**$1,970,930**	**-$266,865**	**-13.5%**
TOTAL DOMESTIC COMPETITIVE	**$2,575,105**	**$2,830,003**	**-$254,899**	**-9.0%**
INTERNATIONAL	**$937,457**	**$954,527**	**-$17,071**	**-1.8%**

Finally, the impact on overall attributable cost per piece is presented in Table 17. The analogous impacts on competitive products are presented in USPS-RM2016-12/NP1. Products for which a relatively large proportion of their total attributable costs are in purchased highway transportation, like Bound Printed Matter Flats, will have a larger percentage reduction in unit attributable cost than a product like Standard Mail High Density and Saturation Letters, which uses relatively little highway transportation.

Table 17

Impact of Capacity to Volume Variabilities on FY 2015 Unit Attributable Costs

	New Attributable Cost Per Piece	FY 2015 Attributable Cost Per Piece	Absolute Change	% Change
FIRST-CLASS				
SINGLE-PIECE LETTERS	$0.267	$0.269	$ (0.0021)	-0.8%
SINGLE-PIECE CARDS	$0.250	$0.251	$ (0.0009)	-0.3%
PRESORT LETTERS	$0.123	$0.124	$ (0.0010)	-0.8%
PRESORT CARDS	$0.079	$0.080	$ (0.0007)	-0.9%
FLATS	$0.919	$0.934	$ (0.0156)	-1.7%
PARCELS	$2.387	$2.430	$ (0.0434)	-1.8%
TOTAL FIRST-CLASS	**$0.203**	**$0.205**	**$ (0.0019)**	**-0.9%**
STANDARD MAIL				
HIGH DENSITY AND SATURATION LETTERS	$0.070	$0.070	$ (0.0002)	-0.3%
HD& SATURATION FLATS & PARCELS	$0.104	$0.105	$ (0.0004)	-0.4%
CARRIER ROUTE	$0.205	$0.206	$ (0.0012)	-0.6%
LETTERS	$0.103	$0.103	$ (0.0007)	-0.6%
FLATS	$0.495	$0.501	$ (0.0058)	-1.2%
PARCELS	$1.456	$1.480	$ (0.0242)	-1.6%
EVERY DOOR DIRECT MAIL - RETAIL	$0.062	$0.062	$ -	0.0%
TOTAL STANDARD MAIL	**$0.137**	**$0.138**	**$ (0.0010)**	**-0.7%**
PERIODICALS				
IN COUNTY	$0.155	$0.155	$ (0.0000)	0.0%
OUTSIDE COUNTY	$0.375	$0.382	$ (0.0072)	-1.9%
TOTAL PERIODICALS	**$0.353**	**$0.360**	**$ (0.0065)**	**-1.8%**
PACKAGE SERVICES				
ALASKA BYPASS	$12.371	$14.935	$ (2.5645)	-17.2%
BOUND PRINTED MATTER FLATS	$0.563	$0.580	$ (0.0172)	-3.0%
BOUND PRINTED MATTER PARCELS	$1.028	$1.048	$ (0.0193)	-1.8%
MEDIA AND LIBRARY MAIL	$4.573	$4.795	$ (0.2218)	-4.6%
TOTAL PACKAGE SERVICES	**$1.309**	**$1.360**	**$ (0.0510)**	**-3.7%**
FREE MAIL - BLIND, HANDICAPPED AND SERVICEMEN	**$0.898**	**$0.915**	**$ (0.0169)**	**-1.8%**
TOTAL DOMESTIC COMPETITIVE	**$2.706**	**$2.775**	**$ (0.0688)**	**-2.5%**

Mathematical Appendix

This Appendix provides the mathematical relationship between the elasticity of capacity and the associated elasticity of capacity utilization. It demonstrates that an assumption of a 100 percent elasticity of capacity with capacity is also an assumption that capacity utilization does not change with changes in volume.

There are three possible cases: capacity changes are proportional to volume changes (100 percent variability), capacity changes are less than proportional to volume changes (less than 100 percent variability, and capacity change are more than proportional to volume changes (greater than 100 percent variability).

To derive the relationship between the two elasticities, suppose that the relationship between capacity (C) and volume (V) is given by:

$$C = \alpha V^{\beta}$$

The marginal change in capacity with respect to volume is given by:

$$\frac{\partial C}{\partial V} = \beta \alpha V^{\beta - 1}$$

The elasticity of capacity with respect to volume is given by:

$$\varepsilon_{C,V} = \frac{\partial C}{\partial V}\frac{V}{C} = \frac{\left(\beta \alpha V^{\beta-1}\right)V}{C} = \frac{\beta \alpha V^{\beta}}{\alpha V^{\beta}} = \beta.$$

Now consider the relationship between this variability and capacity utilization. Define capacity utilization as the ratio of volume to capacity:

$$CU = \frac{V}{C}$$

Now we want to find how capacity utilization responds to volume changes. To do so, we take the derivative of CU with respect to volume.

$$\frac{\partial CU}{\partial V} = \frac{\partial \left(\frac{V}{C}\right)}{\partial V} = \frac{C - \left(\beta \alpha V^{\beta-1}\right)V}{\alpha V^{\beta^2}} = \frac{(1-\beta)}{\alpha V^{\beta}}.$$

We can use this to find the elasticity of capacity utilization with respect to volume:

$$\varepsilon_{CU,V} = \frac{\partial CU}{\partial V}\frac{V}{CU} = \frac{(1-\beta)}{\alpha V^{\beta}}\frac{V}{\left(\frac{V}{C}\right)} = (1-\beta).$$

This last expression can be used to derive the following condition:

If β = 1, then $\varepsilon_{CU,V}$ = 0.

If β < 1, then $\varepsilon_{CU,V} > 0$.

If β > 1, then $\varepsilon_{CU,V} < 0$.